# Reliability of Computer Systems
# Part 1: Basic Reliability Quantification

Peter Sobe

HTW Dresden, Germany
Faculty of Computer Science and Mathematics

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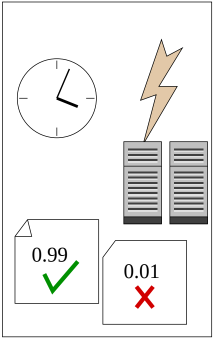
## Overview:

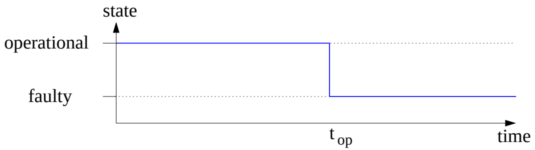
- Probabilities of fault free operation and failures
- Availability of a functionality
- Time aspects:
  - How long is a system operational / faultfree?
  - Mean time to failure, Mean time to repair.
  - Mission time?
- Influence of the system structure / complexity on reliability



# Reliability Measures

**How long does a system operate without failures?**

Expected operation time $E(t_{op})$, MTTF (Mean time to Failure)

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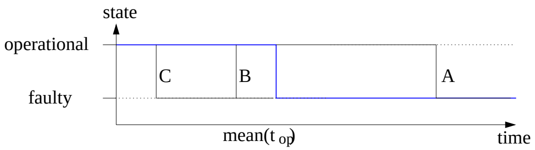
# Reliability Measures

Example: A hard disk in a server is expected to work two years. A clever (?) strategy could be to replace it every two years by a new one.

Important: one should not trust in this period! These two years can be:

- **a mean value**, i.e. when three disks are in operation: 4, 1.5 and 0.5 years. Two out of three devices do not reach the two years :-(
- **a probabilistic measure**, e.g. 99% fail after two years



# Reliability Measures

**How likely is a faultfree operation for a specified period?**

Perspective: L ... lifetime, $t_{op}$, MTTF
At time $t$:

- P(operational at time $t$): $R(t) = P(L > t)$
- $R(t)$ is the probability that the lifetime of the device is longer than the time period t
- $R(t)$ is the probability that the device is faultfree at time $t$

Reliability is the probability of a system performing its purpose adequately for the period of time intended under the operating conditions encountered[1].

$R(t) = P(L > t)$

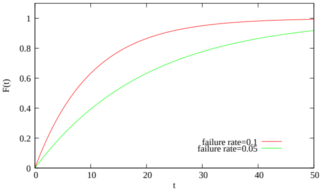
$L$ ... lifetime
$t$ ... time period, or current time when system runs since $t_0 = 0$

[1] From A.L. Reibman, M. Veeraraghavan: Reliability Modeling: An Overview for Systems Designers. In: N. Suri, C.J. Walter, M.M. Hugue. Advances in Ultra-Dependable Distributed Systems, IEEE Computer Society Press, 1995

**Failure probability**

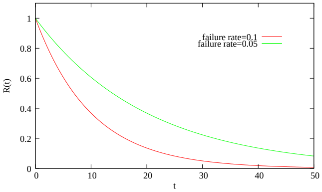
$$F(t) = 1 - e^{-\lambda t}$$

compares to a CDF




**Reliability**

$$R(t) = 1 - F(t)$$
$$R(t) = e^{-\lambda t}$$




Parameter $\lambda$: failure rate, measured in $\frac{1}{time\ unit}$.
For example one failure per 10 years is expressed by $0.1\frac{1}{year}$

**Mean Time to Failure**

Expected value of L:

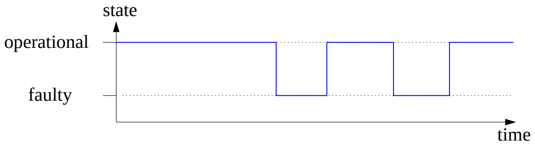
$$MTTF = E(t) = \frac{1}{\lambda}$$

Mean Time to Failure is the expected time until a system fails.

## Availability

Q: How often is a system out of operation during a specified period?

$A$, $A(t)$: Availability is the probability that a system will be operational at a given time.




$A$: stationary case without any assumption on the time

## Availability

Observation of faulty and operational states requires to take repair (or any kind of recovery) into account.

Parameters:

- $\lambda$: failure rate, and MTTF = $\frac{1}{\lambda}$
- $\mu$: repair rate, and MTTR = $\frac{1}{\mu}$

$$A = lim_{t \to \infty} A(t) = \frac{\mu}{\lambda + \mu}$$

$$A = \frac{MTTF}{MTTF + MTTR}$$

# Example (1/2)

## Single component, no repair

Failure rate $\lambda = 0.5 \frac{1}{year}$, MTTF = 2 years
$R(t) = e^{-\lambda t}$

| t | 10 d | 1 month | $\frac{1}{2}$ year | 1 year | 2 years | 3 years | 5 years |
|---|---|---|---|---|---|---|---|
| R(t) | 0.9864 | 0.9592 | 0.7788 | 0.6065 | 0.3679 | 0.2231 | 0.0821 |

These numbers tell us:

- A mission time shorter than the MTTF does not necessarily mean that the system is unlikely to fail.
- A longer operation than the MTTF leads to a very high failure probability
- For a mission time = MTTF:
  $R(MTTF) = R(\frac{1}{\lambda}) = e^{-1} = 0.37$

## Single component, with repair

Failure rate $\lambda = 0.5 \frac{1}{year}$
Repair rate $\mu = 36.5 \frac{1}{year}$ (10 days outage for repair)
MTTR = $\frac{1}{\mu} = \frac{1}{36.5}$ year
MTTF = 2 years

Availability:

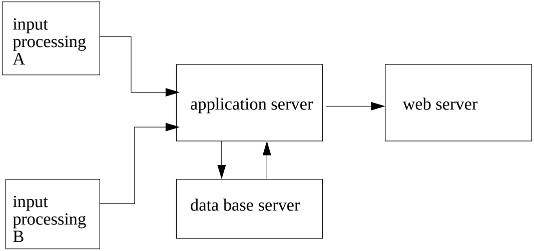
$$
\begin{aligned}
A &= \frac{\mu}{\lambda + \mu} \\
&= \frac{36.5}{0.5 + 36.5} = 0.9865
\end{aligned}
$$

The same result is obtained by

$$
\begin{aligned}
A &= \frac{MTTF}{MTTF + MTTR} \\
&= \frac{2}{2 + \frac{1}{36.5}} = 0.9865
\end{aligned}
$$

# System structure

Complex systems consists of many independent components that all can fail




Effects:

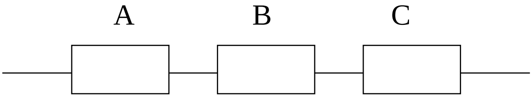
- Complexity increases the susceptibility against component faults
- The reliability of the system decreases with increasing complexity[2]

[2] But this can be compensated by fault-tolerance techniques

# System structure

Complex System: Every component is required to work properly

Serial composition (RBD[3]): compares to an electrical circuit




A failure of any single component causes a system failure

$$R_{total} = R_A \times R_B \times R_C$$
$$F_{total} = 1 - ((1 - F_A) \times (1 - F_B) \times (1 - F_C))$$

[3] Reliability Block Diagram

# System structure: serial composition

Example 1:
Three servers

- database server: R=0.999, F= 0.001
- application server: R=0.995, F = 0.005
- web server: R=0.99, F=0.01

$$\begin{aligned} R &= 0.999 \cdot 0.995 \cdot 0.99 &= 0.984 \\ F & &= 0.016 \end{aligned}$$

Example 2:
Parallel computer, composed of 500 blades (main boards)
$R_{blade} = 0.9999$, $F_{blade} = 0.0001$

$$\begin{aligned} R &= R_{blade}^{500} &= 0.9512 \\ F & &= 0.0488 \end{aligned}$$

# System structure: serial composition

Availability of a serial composition, $N$ components

$$A = \frac{MTTF}{MTTF + MTTR}$$

$$A_{serial} = \prod_{i=1}^{N} A_i$$

$A_i$: availability of component $i$

Uniform system: $A_i = A$ for all $i$, $A_{serial} = A^N$

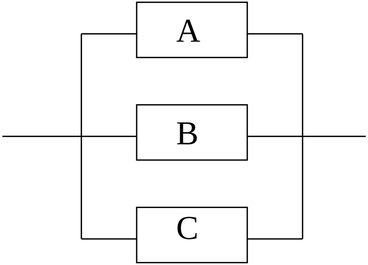
Example:

| A | 0.999 | 0.99 | 0.95 | 0.90 | 0.85 |
|---|---|---|---|---|---|
| N=2 | 0.998 | 0.98 | 0.90 | 0.81 | 0.72 |
| N=3 | 0.997 | 0.97 | 0.85 | 0.73 | 0.61 |
| N=4 | 0.996 | 0.96 | 0.81 | 0.65 | 0.52 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| N=10 | 0.904 | 0.81 | 0.60 | 0.35 | 0.19 |

# System structure: parallel composition

Redundant systems: components are able to substitute each other

Parallel composition (RBD) compared to an electrical circuit




Only failures of all single components cause a system failure:

$$F_{total} = F_A \times F_B \times F_C$$
$$R_{total} = 1 - ((1 - R_A) \times (1 - R_B) \times (1 - R_C))$$

# System structure: parallel composition

Example: A database server that is duplicated
$R_{dbserver} = 0.999$, $F_{dbserver} = 0.001$

$$
\begin{aligned}
F &= F_{dbserver} \cdot F_{dbserver} = 0.001 \cdot 0.001 &&= 0.00001 \\
R &= 1 - F &&= 0.99999
\end{aligned}
$$

# Summary of part 1

- Reliability can be quantified by measures:
  - Failure rate (empiric value)
  - Mean time to failure (MTTF, Expected life time)
  - Reliability (Probability of not failing)
  - Failure probability
  - Availability
- Reliability and failure probability depend on mission time of the system

# Summary of part 1 (cont.)

- Long running or critical computer applications (e.g. server applications) require
  - Either reliable components (with a low failure rate)
  - or repair during operation
  - even proactive replacement of components during operation
- System structure
  - Complexity (i.e. a high number of components that rely on each other) reduces reliability
  - Redundancy (systems that can substitute other) increases reliability